NOTATION

T_i, temperature of the i-th OTC, K; P_{0i}, power of the internal heat generation in the OTC, W; α_i, convective heat-transfer coefficient at the section in contact with the i-th OTC, W/(m²·K); S_i, surface area of convective heat exchange at the segment in contact with the i-th OTC, m²; c, specific heat capacity of the coolant, J/(kg·K); M, mass flow rate of the coolant, kg/sec; T_w, temperature of the conduit wall, K; T_f, temperature of the coolant flow, K; α_{rad}, coefficient of radiative heat exchange between the conduit and the ambient medium, W/(m²·K); U_{rad}, outside perimeter of the conduit, m; L, length of the conduit, m; α_{con}, coefficient of convective heat exchange in the heat exchanger, W/(m²·k); U_{ins}, inside perimeter of the conduit, m; λ, coefficient of thermal conductivity for the material of the conduit, W/(m·K); F, cross-sectional area of the conduit, m².

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THE CONTROL OF QUASIUNIFORM HEATING OF A CYLINDRICAL SPECIMEN IN AN INDUCTOR

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We have used a computer mathematically to experiment with and to formulate solutions for the problem of optimum control by means of quasiuniform heating of Foucault currents in cylindrical steel specimens.

The technology used in the heat treatment of metal specimens requires that they be heated uniformly, within the limits of some tolerance δ, to a specified temperature $\hat{u}$. Such quasiuniform heating, regardless of its source, can either be achieved within some given interval of time, or within a minimum period of time whose estimation is of interest from the standpoint of economic control of the technological processes.

The problems of using quasiuniform heating to achieve control have been examined, in particular, in [1, 2], where the control functions where the temperature of the outside medium for the flow of heat coming from the outside was taken as a function of time.

In this paper, the heating is achieved by means of Foucault currents that are generated within the specimen by means of a high-frequency field from a solenoid inductor into which the specimen has been placed.

Within the framework of the axial-symmetric three-dimensional-uniform model in [3, 4] a method is applied to the problems of annealing steel specimens for purposes of calculating the temperature field generated by such a source.

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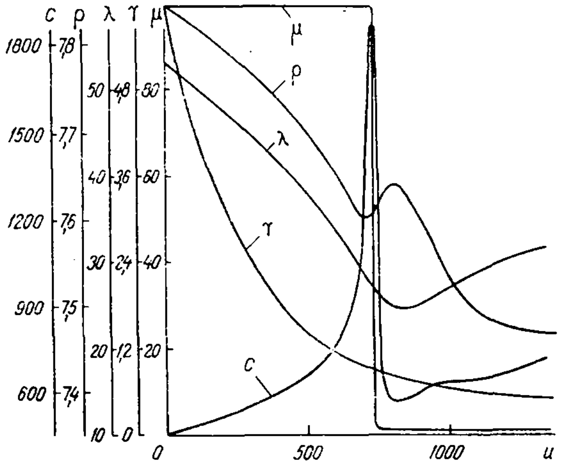

Fig. 1. Heat capacity c, J/(kg·K), density $\rho\cdot 10^3$, kg/m^3, thermal conductivity λ, W/(m·K), specific conductivity $\gamma\cdot 10^6$, 1/(Ω·m), and relative magnetic permeability μ as functions of the temperature u, °C.

We have in mind the heat that is generated in welding, i.e., to temperatures higher than those used in the cited references, and unlike these works, we also necessarily make provision for the radiative exchange of heat at the surface.

1. The induced electromagnetic field (E, H), in analogy with [3, 4], is described in quasisteady approximation and an algorithm is developed to determine the temperature u = u(r, t) of the cylindrical specimens by means of the following system of differential equations:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda(u)\frac{\partial u}{\partial r}\right)+\frac{1}{2}\gamma(u)|E|^2=c(u)\rho(u)\frac{\partial u}{\partial t},\ r\in(0,R),\ t\in(0,T),$$
$$\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r}{\gamma(u)}\frac{\partial H}{\partial r}\right)=i\omega\mu(u)\mu_0 H \tag{1}$$

with the conditions:

$$u|_{t=0}=u_0,\ \left.\frac{\partial u}{\partial r}\right|_{r=0}=0,\ \left.-\lambda(u)\frac{\partial u}{\partial r}\right|_{r=R}=\varkappa(u|_{r=R}-u_0)+\sigma(\theta^4|_{r=R}-\theta_0^4),$$
$$\left.\frac{\partial H}{\partial r}\right|_{r=0}=0,\ H|_{r=R}=nI(t). \tag{2}$$

Here $\theta = 273.15 + u$; $\theta_0 = \theta|_{t=0}$;

$$E(r,t)=\frac{1}{r}\int_0^r i\omega\mu(u)\mu_0 H(r',t)r'dr';$$

n(~100) denotes the number of solenoid winding turns per unit length; I(t) is the amplitude of the current strength in the inductor, i.e., the control function; $\omega(2\pi\cdot 2.5\cdot 10^3$ 1/sec) is the cyclical frequency.

Let us note that the approximation equation for the magnetic field is validated by the condition

$$\left|\frac{dI}{dt}\right|\ll\omega I, \tag{3}$$

whose satisfaction should be verified during the course of the mathematical experiments involved in the search for I(t).

Figure 1 [5] shows the physical parameters of the material, i.e., thermal conductivity λ, heat capacity c, density ρ, specific electrical conductivity γ, and the relative magnetic permeability μ, as functions of temperature.

In order to solve this problem for each given I(t) we make use of an implicit first-order approximation difference scheme for t and a second-order approximation for r [6]. The boundary conditions for these two fields were approximated with second-order accuracy, which is significant in the presence of a skin effect and by taking the radiative exchange of heat into consideration. The solution of the corresponding system of algebraic equations is determined algorithmically by the sweeping method [6] that is used in the iteration cycle [3] in view of the nonlinearity of the problem. The computer program that employs this complex of methods serves as the data unit for the temperature field in solving the problems of control within the framework of variational formulations.

2. In analogy with [2] let us introduce the following significant functional into our examination:

$$\Phi_T(I) \equiv \int_0^R [u(r,\ T,\ I(t)) - \hat{u}]^2\, r\, dr, \tag{4}$$

where u(r, T, I(t)) is the temperature field at a finite instant of heating time T, generated by the control function I(t), while $\hat{u}$ is the required level of heating.

Let δ be the nonuniformity tolerance, so that the sought function I(t) must satisfy the condition $\Phi_T(I) \le \delta$.

The possible instability in the variational problems [7], leading to the solutions, is virtually unattainable, requires consideration within the formulation of the conditions of realization, one of which, in the problem under consideration, is the limitation on the current strength: $0 \le I(t) \le I_{max}$. Let us note: if condition (3) is also explicitly included in the formulation of the problem, then the limitation on the current strength means that a limitation is also imposed on its derivative, which ensures the compactness of the set {I(t)} within a class of continuous functions, which means [7] that the problem of finding I(t) is stable.

Let us examine the various aspects of the mathematical formulation of the control problem.

The problem of seeking out the control function I(t), which, at the given instant of time T, within the limits of the tolerance δ, provides a uniform temperature field, can be formulated as a variational problem with respect to the set, having the following limitations:

$$\Phi_T(I) \leqslant \delta,\ I(t) \in K \equiv \left\{ I(t) \in C[0,\ T] : 0 \leqslant I(t) \leqslant I_{\max},\ \left| \frac{dI}{dt} \right| \leqslant \varepsilon\omega I \right\}, \tag{5}$$

where $\varepsilon \ll 1$ is some given parameter.

We note that there may be no solution to problem (5) because the parameters δ, I_{max}, and T are, in actual practice, given as independent of each other. In this event, where the value of I_{max} for the power supply is not overly large may be inadequate for purposes of heating to a temperature $\hat{u}$ within the given time T; this situation becomes even more difficult because the presence of the skin effect raises the nonuniformity of the field, so that at the a priori given instant of time T we may have $\Phi_T(I) > \delta$. We will proceed on the basis that the parameters have been coordinated so that there exists at least one solution to problem (5) (the problem is consistent [7]).

The possible nonidentity of the solution for a finite δ is insignificant from the standpoint of the control problem. However, stability is assured through the choice of the set K.

A solution can be found for problem (5) by making use of the familiar methods of minimizing the functional Φ_T on the set K [8], so as to carry out some iteration process with respect to the given initial approximation. However, in actual practice it is more convenient to use a different formulation which makes indirect provision for the limitation imposed on the derivative of the current. Specifically, let us introduce the conditional stabilizer [7, 9]:

$$\Omega(I) \equiv \int_0^T (I'(t))^2\, dt,\ I'(0) = 0,\ I(T) = 0.$$

In this event, the following variational problem is correct under the consistency condition:

$$(I(t) = \arg\inf \Omega(I),\ \ I(t) \in K_\delta \equiv \{I(t) \in C^1(0,\ T);\ \Phi_T(I) \leqslant \delta,\ \ 0 < I(t) \leqslant I_{\max},$$
$$I'(0) = 0,\ I(T) = 0\}. \tag{6}$$

The algorithm for the solution of this problem will be dealt with in the following section.

Problem (6) [or (5)] may be treated as an element of another problem, which deals with the determination of the minimum time T^* and the corresponding control function $I^*(t)$ under all the previous conditions imposed on the temperature field. This problem (the problem of rapid action) is formulated in analogy with [2] for the functional $T(I)$, implicitly defined by conditions (6):

$$T^* = T(I^*) = \inf T(I),\ \ I(t) \in K_\delta. \tag{7}$$

An approximate solution for this problem can be found [2] on a grid of values $\{T_s\}$, $s = 1, 2, \ldots, m$, $T_{s+1} < T_s$, at each point of which problem (6) is solved for I_s. As a result of the solution of this last problem, in the case of sequential s we obtain

$$\Phi^*(T_s) = \Phi_{T_s}(I_s).$$

Numerical experimentation reveals a natural monotonic increase in $\Phi^*(T)$ with a reduction in T, and to the extent that with a sufficiently large T, $\Phi^*(T) < \delta$, there exists a root of the equation $\Phi^*(T) = \delta$. This root T^*, approximated on the introduced grid, is obviously the sought minimum value, and the corresponding solution of problem (6) is the sought control $I^*(t)$, $t \in [0, T^*]$.

It is equally obvious, in turn, that the values $[T^*, I^*(t)]$ can be refined by utilization of a more exact method for the solution of the equation $\Phi^*(T) = \delta$ with an algorithmically determined left-hand side.

Problems (5) and (7) can be markedly simplified, if we look for the control on the set of constant current-amplitude values: $I(t) \equiv I = \text{const}$, $t \in [0, T]$. In this case, condition (3) is automatically satisfied, and the magnitude of the control currents which provides for the quasiuniform heating over the given interval of time is defined by the problem

$$\Phi_T(I) \leqslant \delta,\ \ I \in K_0 \equiv \{I \in R^{(1)} : 0 < I \leqslant I_{\max}\}. \tag{5'}$$

For the solution of this problem it is enough to find the minimum Φ_m of the function $\Phi_T(I)$ on $[0, I_{max}]$. If $\Phi_m \leq \delta$, the corresponding value of I_m is the solution; in the opposite case, problem (5') is inconsistent and the parameters I_{max}, δ, and T must be corrected.

Let us also examine the problem of determining the optimum heating time with a control analogous to (5'):

$$T^* = \inf T(I),\ \ I \in K_0. \tag{7'}$$

The method for its solution does not differ from the one described above for problem (7), and it is only the functional $T(I)$ that is implicitly determined by conditions (5').

3. Solution of problem (6) can be found by means of the general regularizing Tikhonov operator [7]. In this case, the sought $I(t)$ can be chosen from the sequence of extrema of the smoothing functional $F_{\alpha_p}(I) = \Phi_T(I) + \alpha_p(\Omega)(I)$, $\alpha_p \to 0$, which is minimized for each α^p on the set

$$\tilde{K} \equiv \{I(t) : 0 < I(t) \leqslant I_{\max},\ I'(0) = 0,\ I(T) = 0\}.$$

If $I_{\alpha_p}(t)$ is the extremum, the required approximation $I_{\bar{\alpha}}(t)$ is determined by the condition $\bar{\alpha} = \max\limits_{M} \alpha_p$, $M \equiv \{I_{\alpha_p} : \Phi_T(I_{\alpha_p}) \leqslant \delta\}$. With the chosen stabilizer [9]

$$\lim_{\alpha_p \to \infty} I_{\alpha_p}(t) = 0,\ \lim_{\alpha_p \to \infty} F_{\alpha_p}(I_{\alpha_p}) = \Phi_T(0) \equiv \int_0^R (u_0 - \hat{u})^2\, r dr.$$

Therefore, and because of the monotonic diminution of $\varphi(\alpha_p) = \Phi_T(I_{\alpha_p})$ as $\alpha_p \to 0$, we can use the described algorithm if $\delta < \Phi_T(0)$ (in the opposite case $u_0 \approx \hat{u}$ within the limits of the

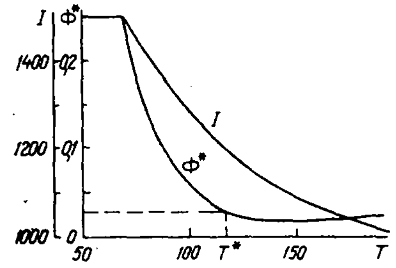

Fig. 2

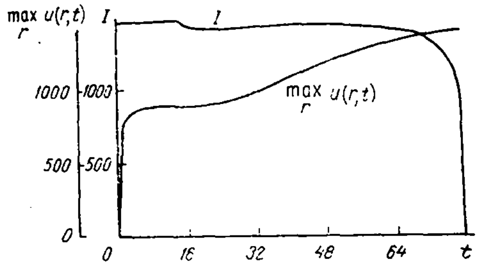

Fig. 3

Fig. 2. The optimum values of $\Phi^*(T)$ and control I, A, as functions of the process time T, sec, in problem (5').

Fig. 3. Solution of problem (7) I(t), A, and the corresponding $\max_r u(r, t)$, °C, as functions of the time t, sec.

given error the problem loses sense). On the other hand [9], this algorithm is convenient from the standpoint that for the initial approximation to find I_{α_p} we can take the earlier found $I_{\alpha_{p-1}}$, and if α_0 is sufficiently large, the initial starting point for the determination of I_{α_p} will be $I(t) \equiv 0$.

We used the method of gradient projection [1] (on the set $\tilde{K}$) to minimize the functionals in all of the cases. For problem (6) its simplest modification is described by the formula

$$I_{k+1}(t)=\begin{cases} I_k-\nu_k F'_{\alpha_p}(I_k) & \text{when } 0<I_{k+1}<I_{\max},\\ 0 & \text{when } I_{k+1}\leqslant 0,\\ I_{\max} & \text{when } I_{k+1}\geqslant I_{\max}, \end{cases} \tag{8}$$

where $F_{\alpha_p}'(I_k)$ is the value of the gradient of the corresponding functional, and ν_k is chosen on the basis of the condition:

$$\nu_k=\arg\inf F_{\alpha_p}(I_k-\nu F'_{\alpha_p}(I_k)),$$

which corresponds to the "steepest descent" [1]. We can be assured that

$$F'_\alpha(I)=\Phi'_T(I)-2\alpha I''(t),$$

where for the second derivative of the current we employ the finite-difference second-order approximation of accuracy. The gradient Φ_T' of the functional Φ_T replaces F_{α_p}' in formula (8) for problem (5').

The value of $\Phi_T'(I_k)$ is determined algorithmically by means of the boundary-value problem [1, 10] conjugate to (1)-(2). In this case, it is convenient to utilize the linearization of the latter, referring to the temperature-dependent coefficients to the previous iteration. Let the temperature increment Δu correspond to the increment $\Delta I(t)$. Then

$$\Delta\tilde{\Phi}_T=2\int_0^R[\tilde{u}(r,T;I)-\hat{u}]\,\Delta u(r,T)\,rdr+\int_0^R[\Delta u(r,T)]^2rdr\equiv V_1+V_2.$$

In view of the linearization

$$V_1=\int_0^T\int_0^R\gamma(u_k)|E|^2\psi(r,t)\,\Delta I/I_k\,rdrdt,$$

where $\psi(r, t)$ is the solution of the conjugate boundary-value problem:

$$\lambda(u_k)\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\psi}{\partial r}\right)+c(u_k)\rho(u_k)\frac{\partial\psi}{\partial t}=0,\ T>t\geqslant 0,\ 0<r<R,$$

$$\psi|_{t=T}=\frac{2(u_k(r,\ T)-\hat{u})}{c(u_k)\rho(u_k)}, \tag{9}$$

$$\left.\frac{\partial\psi}{\partial r}\right|_{r=0}=0,\quad -\lambda(u_k)\left.\frac{\partial\psi}{\partial r}\right|_{r=R}=\psi V(u_k)|_{r=R},$$

$$V(u_k)=\varkappa+4\sigma(u_k+273.15)^3.$$

This problem is solved in parallel with (1)-(2) with the aid of the analogous difference scheme.

As a consequence of the upper and lower bounds of the physical parameter-coefficients in (1)-(2) for V_2 we obtain the estimate:

$$V_2\leqslant C\int_0^T(\Delta I)^2\,dt\equiv C\,\|\Delta I\|_{L_2}^2,$$

where the constant C is independent of ΔI.

Thus, $\Delta\tilde{\Phi}_T = (B,\ \Delta I)_{L_2} + O(\|\Delta I\|_{L_2}{}^2)$ and by definition [11] we differentiate the approximate functional Φ_T

$$\Phi'(I_k)\equiv B=\int_0^R\gamma(u_k)\,|E|^2\,\psi(r,\ t)/I_k(t)\,r\,dr. \tag{10}$$

4. Some of the results from the solution of the control problems for the heating of steel specimens, as presented below, pertain to St40 steel with the following values for the parameters of the problem: u_0 = 20°C, $\hat{u}$ = 1400°C, $\sigma = 0.7\sigma_0$, $\kappa = 0$, $\delta = 0.0288$, I_{max} = 1500 A, where σ_0 is the Stefan–Boltzmann constant.

We see from Fig. 2 that T*, corresponding to the solution of problem (7'), is equal to 117 sec. In this case, I* = 1193 A. We should also take note of the possibility of a more uniform heating of the specimen by a current of constant amplitude, where $\delta = \delta_m = 0.0167$. This corresponds to a process duration of T = 149 sec, and a current strength of I = 1089 A.

Figure 3 shows the results for the solution of the high-speed problem (7). The relationship between the amplitude of the control current and $\max_r u(r,\ t)$ corresponds to the minimum T* = 79 sec.

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